Inferring physical conditions in star forming regions with new Bayesian approach and spatial regularization

Point of the presentation

Effect of **radiative feedback** on parent cloud

Efficient estimation of **physical conditions** in **star forming region**

Carina nebula

Pillars of creation

Source: [https://www.nasa.gov/](https://www.nasa.gov/)
Example: the OrionB cloud

Observations led by OrionB Consortium, (Pety et al., 2016)

Very rich dataset!
\[ \approx 10^6 \text{ pixels} \]
\[ \approx 100 \text{ pc}^2 \]
tens of lines

Great variations of SNR
\[ \implies \text{Between pixels} \]
\[ \implies \text{Between lines} \]

Source: IRAM-30m, J.Pety, M. Guerin, consortium Orion-B
Estimation procedure

Physical Parameters

\( T, C_0, n_H, \zeta, \ldots \)

(Grid of)

Physical model predicts

Predicted observations

Updates

Compares With likelihood or least squares

Actual observations

observations = Integrated intensities of molecular / atomic lines

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Carina nebula (Wu et al., 2018)

Pixel-wise estimation: reconstructs well observations
Carina nebula (Wu et al., 2018)

Pixel-wise estimation: reconstructs well observations

But reconstructed maps have no spatial coherence particularly in low SNR regions
Example: the OrionB cloud

For better estimations:
⇒ Spatial regularization
⇒ uncertainty quantification
⇒ Fast

Method evaluated on a synthetic PDR map

Source: IRAM-30m, J.Pety, M. Guerin, consortium Orion-B
Photo-Dissociation Region (PDR)

- Atomic layer
- Molecular layer
- Atomic layer

\[ G_0 \]

- Constant \( P_{th} \)
- Total \( A_V \)

- \( H^+ \)
- \( H \)
- \( H_2 \)
- \( C^+ \)
- \( C \)
- \( CO \)
- \( C \)
- \( H \)
- \( H^+ \)

radio - Far Infrared

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Meudon PDR code

(Le Petit et al., 2006)

**Parameters:**
- $P_{th}$ - thermal pressure
- $G_0$ - intensity of radiative field (in Habing units)
- $A_V$ - visual extinction

**Code:**
Iteratively Solves large set of partial differential equations

**Observation:**
Integrated intensities of a line
Synthetic PDR map

true values of $P_{th}$

true values of $G_0$

true values of $A_V$
Synthetic PDR map

True maps
true values of $P_{th}$

Meudon PDR + Noise
true values of $G_0$

Herschel-like Observations
CO (4-3)

CO (13-12)

With previous method
MLE of $G_0$

Proposed method
Synthetic PDR map

True maps
true values of $P_{th}$

Meudon PDR + Noise
true values of $G_0$

Herschel-like Observations

With previous method
MLE of $G_0$

Proposed method
MMSE of $G_0$

CO (4-3)

CO (13-12)
Bayesian approach

Inference with **credibility intervals**
More complete description of parameter space

\[
\mathbb{P}[(P_{th}, G_0, A_V) | Y] \propto \mathbb{P}[Y | (P_{th}, G_0, A_V)] \mathbb{P}[(P_{th}, G_0, A_V)]
\]

A posteriori \hspace{2cm} Likelihood \hspace{2cm} Spatial prior

Complex distribution
\[\Rightarrow\] impossible to extract estimators as is
\[\Rightarrow\] sample from it with **MCMC algorithm**
Observation model

\[
\begin{cases}
\text{If detected: } y_{n,\ell} = \epsilon_{n,\ell}^{(m)} f_{\ell}(\theta_n) + \epsilon_{n,\ell}^{(a)} \\
\text{Otherwise: } y_{n,\ell} \leq \omega
\end{cases}
\]

With

\[y_{n,\ell}: \text{actual observation}\]
\[f_{\ell}(\theta_n): \text{predicted observation}\]
\[\epsilon_{n,\ell}^{(m)}: \text{multiplicative noise}\]
\[\epsilon_{n,\ell}^{(a)}: \text{additive noise}\]
\[\omega: \text{telescope detectability limit}\]

How to deal with \textbf{expensive numerical code} \(f\)?
- Approximation of \(f\) (e.g., with a neural network)

How to deal with both additive and multiplicative noises?
- Approximation of likelihood

<table>
<thead>
<tr>
<th>SNR</th>
<th>Noise model</th>
</tr>
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<tbody>
<tr>
<td>Low</td>
<td>Additive</td>
</tr>
<tr>
<td>High</td>
<td>Multiplicative</td>
</tr>
<tr>
<td>Intermediate</td>
<td>Mixture</td>
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Spatial Regularization

- Neighbour of
- Should be close to the average of

\[ \approx \text{adaptive stacking} \]
Spatial Regularization

Not smooth $\implies$ High penalty

$\Delta$ neighbour of

Should be close to the average of

$\approx$ adaptive stacking
Spatial Regularization

Neighbour of

Should be close to the average of

\(\approx\) adaptive stacking

Not smooth \(\implies\) High penalty \(\times\)

Smooth \(\implies\) Low penalty \(\checkmark\)
Markov Chain Monte Carlo

Some Gaussian Mixture
How to explore all modes?
Monte Carlo Markov Chain

Some Gaussian Mixture
How to explore all modes?

PMALA transition kernel
Fast local exploration

$P_{th}$

$G_0$

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Monte Carlo Markov Chain

Some Gaussian Mixture
How to explore all modes?

PMALA transition kernel
Fast local exploration

MTM transition kernel
Jumps between modes

Pressure $P_{th}$

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$G_0$
Illustration on Gaussian mixture
Synthetic PDR map

True maps
true values of $P_{th}$

Meudon PDR + Noise
true values of $G_0$

Herschel-like Observations

CO (4-3)

With previous method
MLE of $G_0$

Proposed method
MMSE of $G_0$
Application to real data: NGC 7023

One of the most studied PDR
Close to us ( ~ 320 pc)
Edge-on geometry
Illuminated by a spectroscopic binary system (B3Ve and B5)

Previous estimations:

$G_0 \approx 2600$ (Habing units) (Chokshi et al., 1988)
$P_{th} \approx 10^8$ K cm$^{-3}$ (Joblin et al., 2018)
$A_V^{tot} \approx 10$ mag (Joblin et al., 2018)
Sampling results

From Literature
Reconstruction of observations

Line integrated intensity

- High J CO lines
- H2 rovibrational lines
- CH+ lines
Conclusion

- New tool ready to be used! \(\Rightarrow\) will be available on ism.obspm.fr
- **Spatial regularization** \(\rightarrow\) enforces spatial smoothness
- **Bayesian approach** \(\rightarrow\) provides both estimates and credibility intervals

- Applications:
  - synthetic case with 4096 pixels
  - NGC 7023 (1 pixel)
  - Real data (Horse Head, \(\simeq 10^3\) pixels / Orion B \(\simeq 10^6\) pixels)
  - JWST observations
Appendix
Some formulae

Spatial prior

\[ \| \Delta \Theta_d \|_2^2 = \sum_{n=1}^{N} \sum_{i \in V_n} (\theta_{n,d} - \theta_{i,d})^2 \]

Likelihood approximation

\[ \tilde{\pi}(y_n, \ell | \theta_n) \propto \left[ \pi^{(a)}(y_n, \ell | \theta_n)^{1-\lambda(\theta_n)} \pi^{(m)}(y_n, \ell | \theta_n)^{\lambda(\theta_n)} \right]^{1-c_{n,\ell}} \times \left[ F^{(a)}(\omega | \theta_n)^{1-\lambda(\theta_n)} F^{(m)}(\omega | \theta_n)^{\lambda(\theta_n)} \right]^{c_{n,\ell}} \]
Classic observations: $10^0 - 10^2$ pixels

(Joblin et al., 2018)

(Wu et al., 2018)
Wider maps: towards $10^3 - 10^6$ pixels

Source: https://www.esa.int/ESA_Multimedia/Sets/Webb_First_Images/(result_type)/images

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Degeneracies and multimodality

Saturation of CO lines with AV

Multimodality with HCO+

Identical observations